JEE(Advanced) – 2017 TEST PAPER WITH ANSWER

(HELD ON SUNDAY 21st MAY, 2017)

PART-III : MATHEMATICS

SECTION–1 : (Maximum Marks : 28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :

Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, Provided NO incorrect option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

- for example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened

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37. Which of the following is(are) NOT the square of a 3 × 3 matrix with real entries ?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Ans. (A,B)

38. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k ?

(A) p = 5, h = 4, k = –3 (B) p = –1, h = 1, k = –3

(C) p = –2, h = 2, k = –4 (D) p = 2, h = 3, k = –4

Ans. (D)

39. Let a, b, x and y be real numbers such that a – b = 1 and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ?

(A) $-1-\sqrt{1-y^2}$ (B) $1+\sqrt{1+y^2}$ (C) $1-\sqrt{1+y^2}$ (D) $-1+\sqrt{1-y^2}$

Ans. (A,D)

40. Let X and Y be two events such that $P(X)=\frac{1}{3}$, $P(X\mid Y)=\frac{1}{2}$ and $P(Y\mid X)=\frac{2}{5}$. Then

(A) $P(X'\mid Y)=\frac{1}{2}$

(B) $P(X\cap Y)=\frac{1}{5}$

(C) $P(X\cup Y)=\frac{2}{5}$

(D) $P(Y)=\frac{4}{15}$

Ans. (A,D)

41. Let [x] be the greatest integer less than or equal to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous ?

(A) $x = -1$ (B) $x = 0$

(C) $x = 2$ (D) $x = 1$

Ans. (A,C,D)

42. If $2x - y + 1 = 0$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{16}=1$, then which of the following CANNOT be sides of a right angled triangle ?

(A) 2a, 4, 1 (B) 2a, 8, 1

(C) a, 4, 1 (D) a, 4, 2

Ans. (B,C,D)

43. Let $f:\mathbb{R}\to(0,1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?

(A) $e^x-\int_0^x f(t)\sin t\,dt$ (B) $x^9 - f(x)$

(C) $f(x)+\int_0^{\frac{\pi}{2}} f(t)\sin t\,dt$ (D) $x-\int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$

Ans. (B,D)

SECTION–2 : (Maximum Marks : 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

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44. The sides of the right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

Ans. 6

45. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ?

Ans. 2

46. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Ans. 1

47. Words of the length formed using letters A,B,C,D,E,F,G,H,I,J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, y/9x=

Ans. 5

48. Let $f : R \to R$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\frac{\pi}{2}} \left[f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t \, f(t) \right] dt$$

for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x \to 0} g(x) =$

Ans. 2

SECTION–3 : (Maximum Marks : 18)

- This section contains **SIX** questions of matching type.
- This section contains **TWO** tables (each having 3 columns and 4 rows)
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : –1 In all other cases

Column 1,2 and 3 contain conics, equation of tangents to the conics and points of contact, respectively.

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

49. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only **CORRECT** combination ?

(A) (II) (iii) (R) (B) (IV) (iv) (S) (C) (IV) (iii) (S) (D) (II) (iv) (R)

Ans. (D)

50. If a tangent to a suitable conic (Column 1) is found to be y = x + 8 and its point of contact is (8,16), then which of the following options is the only **CORRECT** combination ?

(A) (III) (i) (P) (B) (III) (ii) (Q) (C) (II) (iv) (R) (D) (I) (ii) (Q)

Ans. (A)

51. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1,1), then which of the following options is the only **CORRECT** combination for obtaining its equation ?

(A) (II) (ii) (Q) (B) (III) (i) (P) (C) (I) (i) (P) (D) (I) (ii) (Q)

Ans. (D)

Let $f(x) = x + \log_e x - x \log_e x$, $x \in (0,\infty)$.

* Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
* Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
* Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1,e^2)$	(i) $\lim_{x\to\infty} f(x) = 0$	(P) f is increasing in $(0,1)$
(II) $f'(x) = 0$ for some $x \in (1,e)$	(ii) $\lim_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e,e^2)
(III) $f'(x) = 0$ for some $x \in (0,1)$	(iii) $\lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in $(0,1)$
(IV) $f''(x) = 0$ for some $x \in (1,e)$	(iv) $\lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e,e^2)

52. Which of the following options is the only **CORRECT** combination ?

(A) (IV) (i) (S) (B) (I) (ii) (R) (C) (III) (iv) (P) (D) (II) (iii) (S)

Ans. (D)

53. Which of the following options is the only **CORRECT** combination ?

(A) (III) (iii) (R) (B) (I) (i) (P) (C) (IV) (iv) (s) (D) (II) (ii) (Q)

Ans. (D)

55. Which of the following option is the only **INCORRECT** combination ?

(A) (II) (iii) (P) (B) (II) (iv) (Q) (C) (I) (iii) (P) (D) (III) (i) (R)

Ans. (D)